Biaxial Flexure Capacity Verification

Description: Calculation of flexure capacity use strain compatibility of a section bending bi-axially. Stress strain curve for concrete comes from Collins and Mitchell.

Reference: PCI Design Handbook 8th Edition|
ACI 318-14
Collins, Michael P. and Mitchell, Denis, Prestressed Concrete Structures, Prentice Hall, Englewood Cliffs, NJ, 1991.

Concrete Geometry and Material Properties

Elastic modulus of precast section $E_c := 4286\ \text{ksi}$

Compressive strength of precast section $f_c := 5000\ \text{psi}$

Ultimate compressive strain in concrete $\varepsilon_{cu} := 0.003$

Constant used for stress strain curve of concrete $n := \dfrac{E_c}{E_c - \dfrac{f_c}{0.003}} = 1.6363$

Width of the stem $w_{stem} := 6\ \text{in}$

Height of the stem $h_{stem} := 15.5\ \text{in}$

Width of the ledge $w_{ledge} := 36\ \text{in}$

Thickness of the ledge $t_{ledge} := 3.5\ \text{in}$

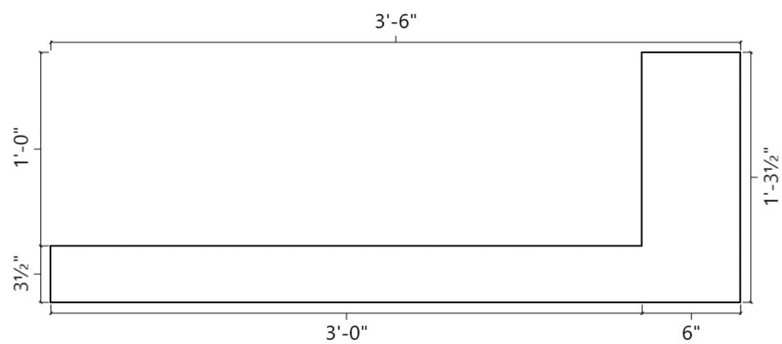


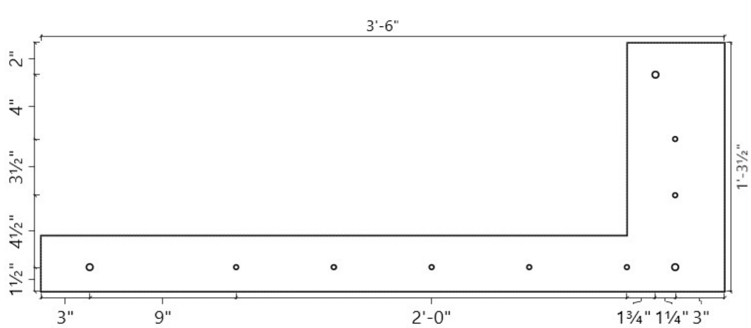
Figure 1: Concrete Geometry of Composite Precast Riser Section

Reinforcement Quantities and Properties

Yield strength of rebar $f_y := 60\ \text{ksi}$

Yield strength of mesh $f_m := 65\ \text{ksi}$

Elastic modulus of mild reinforcement $E_s := 29000\ \text{ksi}$



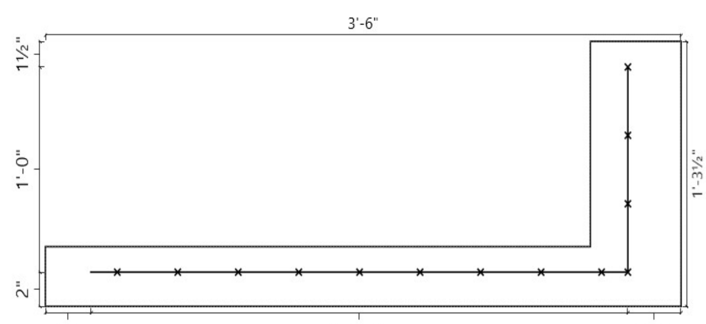


Figure 2: Reinforcement Locations of Strand (Left) and Rebar (Right)

Location and area of all rebar

$$x_{bars} := \begin{bmatrix} 3 & 1.5 \\ 12 & 1.5 \\ 18 & 1.5 \\ 24 & 1.5 \\ 30 & 1.5 \\ 36 & 1.5 \\ 39 & 1.5 \\ 39 & 6 \\ 39 & 9.5 \\ 37.75 & 13.5 \end{bmatrix} \text{in} \qquad A_{bars} := \begin{bmatrix} 0.6 \\ 0.31 \\ 0.31 \\ 0.31 \\ 0.31 \\ 0.31 \\ 0.6 \\ 0.31 \\ 0.31 \\ 0.6 \end{bmatrix} \text{in}^2$$

Location and area of all wires

$$x_{wires} := \begin{bmatrix} 4.75 & 2 \\ 8.75 & 2 \\ 12.75 & 2 \\ 16.75 & 2 \\ 20.75 & 2 \\ 24.75 & 2 \\ 28.75 & 2 \\ 32.75 & 2 \\ 36.75 & 2 \\ 38.5 & 2 \\ 38.5 & 6 \\ 38.5 & 10 \\ 38.5 & 14 \end{bmatrix} \text{in} \qquad A_{wire} := 0.04 \text{ in}^2$$

Flexure Parameters

Neutral axis depth $\qquad c := 5.60444 \text{ in}$

Neutral axis angle $\qquad \theta := 14.52487 \text{ deg}$

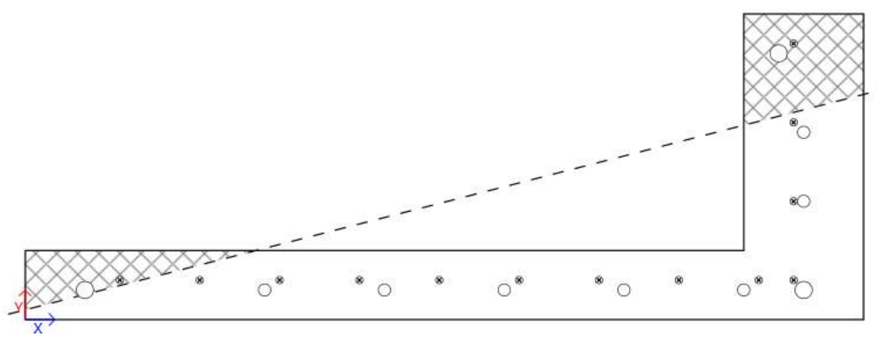


Figure 3: Neutral axis shown on section. Hatched section is the area in compression

Concrete Force

The compression fiber is located at the top left corner of the stem on the riser. The neutral axis is placed relative to this location.

Depth of the compression block $\qquad a := 0.80 \cdot c = 4.4836 \text{ in}$

Distance from top of stem to neutral axis on left edge $\qquad y_{ls} := \dfrac{a}{\cos(\theta)} = 4.6316 \text{ in}$

Change in neutral axis height across stem width $\qquad y_{rs} := w_{stem} \cdot \tan(\theta) = 1.5545 \text{ in}$

The compression block on the stem is split into a rectangular portion and a triangular portion. This is done to make the centroid computation easier to due by hand.

Area of the rectangular portion of the compression block $A_{sr} := w_{stem} \cdot \left(y_{ls} - y_{rs}\right) = 18.4626\ in^2$

Centroid in y direction of rectangular block $cy_{sr} := h_{stem} - \left(\frac{y_{ls} - y_{rs}}{2}\right) = 13.9615\ in$

Centroid in x direction of rectangular block $cx_{sr} := w_{ledge} + \frac{w_{stem}}{2} = 39\ in$

Area of the triangular portion of the comrpression block $A_{st} := \frac{y_{rs} \cdot w_{stem}}{2} = 4.6635\ in^2$

Centroid in y direction of triangular block $cy_{st} := h_{stem} - y_{ls} + \frac{2}{3} \cdot y_{rs} = 11.9047\ in$

Centroid in x direction of rectangular block $cx_{st} := w_{ledge} + \frac{w_{stem}}{3} = 38\ in$

Total area of the stem's compression block $A_{stem} := A_{sr} + A_{st} = 23.126\ in^2$

Centroid in y direction of the stem's compression block $cy_{stem} := \frac{A_{sr} \cdot cy_{sr} + A_{st} \cdot cy_{st}}{A_{stem}} = 13.5467\ in$

Centroid in x direction of the strem's compression block $cx_{stem} := \frac{A_{sr} \cdot cx_{sr} + A_{st} \cdot cx_{st}}{A_{stem}} = 38.7983\ in$

Elevation of netural axis on left edge of the flange $y_f := h_{stem} - y_{ls} - w_{ledge} \cdot \tan(\theta) = 1.5415\ in$

Distance of neutral axis from elft edge of the flange $x_f := \frac{t_{ledge} - y_f}{\tan(\theta)} = 7.5594\ in$

Total area of the flange's compression block $A_{flange} := \frac{1}{2} \cdot x_f \cdot \left(t_{ledge} - y_f\right) = 7.4025\ in^2$

Centroid in y direction of flange's compression block $cy_{flange} := t_{ledge} - \frac{t_{ledge} - y_f}{3} = 2.8472\ in$

Centroid in x direction of flange's compression block $cx_{flange} := \frac{x_f}{3} = 2.5198\ in$

Area of compression block $A_c := A_{stem} + A_{flange} = 30.5285\ in^2$

Centroid in x direction of compression block $cx_c := \frac{A_{stem} \cdot cx_{stem} + A_{flange} \cdot cx_{flange}}{A_c} = 30.0016\ in$

Centroid in y direction of compression block $cy_c := \frac{A_{stem} \cdot cy_{stem} + A_{flange} \cdot cy_{flange}}{A_c} = 10.9523\ in$

Total concrete force $F_c := 0.85 \cdot f_c \cdot A_c = 129.7462\ kip$

Internal moment about the horizontal moment $M_{xc} := F_c \cdot cy_c = 118.4184\ kip\ ft$

Internal moment about the vertical moment $M_{yc} := -\left(F_c \cdot cx_c\right) = -324.383\ kip\ ft$

Mild Force

The neutral axis, as a line, is shifted to the compression fiber. With this definition we can compute the depth of the reinforcement as the distance from this line.

Y Intercept of compression fiber line $b := h_{stem} - w_{ledge} \cdot \tan(\theta) = 6.1731\ in$

Slope of compression fiber line $m := \tan(\theta) = 0.2591$

The following parameters are computed using simple programing. The equations being used are shown below with results of the equations following afterwards.

Depth of each rebar

for $i := 1, i \le 10, i := i + 1$

$$d_{bars_i} := \frac{\left| m \cdot x_{bars_{i\,1}} - x_{bars_{i\,2}} + b \right|}{\sqrt{m^2 + 1}}$$

Strain of each rebar

for $i := 1, i \le 10, i := i + 1$

$$\varepsilon_{bars_i} := \frac{c - d_{bars_i}}{c} \cdot \varepsilon_{cu}$$

Stress in the concrete if bar is in compression

for $i := 1, i \le 10, i := i + 1$

$\sigma_{c.bars_i} :=$ if $\varepsilon_{bars_i} < 0$

0 ksi

else

$$f_c \cdot \frac{n \cdot \frac{\varepsilon_{bars_i}}{0.003}}{n - 1 + \left(\frac{\varepsilon_{bars_i}}{0.003} \right)^n}$$

Stress in each bars less concrete stress

for $i := 1, i \le 10, i := i + 1$

$$\sigma_{bars_i} := \max\left(\left[\left[E_s \cdot \varepsilon_{bars_i} \quad -f_y \right] \right] \right) - \sigma_{c.bars_i}$$

Force in each bar

for $i := 1, i \le 10, i := i + 1$

$$F_{bars_i} := A_{bars_i} \cdot \sigma_{bars_i}$$

Arrays of computed values using the above equations

$$d_{bars} = \begin{bmatrix} 5.276 \\ 7.533 \\ 9.038 \\ 10.543 \\ 12.048 \\ 13.553 \\ 14.305 \\ 9.949 \\ 6.561 \\ 2.375 \end{bmatrix} \text{in} \qquad \varepsilon_{bars} = \begin{bmatrix} 0.0002 \\ -0.001 \\ -0.0018 \\ -0.0026 \\ -0.0034 \\ -0.0043 \\ -0.0047 \\ -0.0023 \\ -0.0005 \\ 0.0017 \end{bmatrix} \qquad \sigma_{c.bars} = \begin{bmatrix} 0.742 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 4.524 \end{bmatrix} \text{ksi}$$

$$\sigma_{bars} = \begin{bmatrix} 4.354 \\ -29.943 \\ -53.303 \\ -60 \\ -60 \\ -60 \\ -60 \\ -60 \\ -14.843 \\ 45.608 \end{bmatrix} \text{ksi} \qquad F_{bars} = \begin{bmatrix} 2.613 \\ -9.282 \\ -16.524 \\ -18.6 \\ -18.6 \\ -18.6 \\ -36 \\ -18.6 \\ -4.601 \\ 27.365 \end{bmatrix} \text{kip}$$

Total moment the rebar is generating about the x axis

$$M_{bx} := \sum_{i=1}^{10} x_{bars_{i\,2}} \cdot F_{bars_i} = 3.4684 \text{ kip ft}$$

Total moment the rebar is generating about the y axis

$$M_{by} := \sum_{i=1}^{10} -x_{bars_{i\,1}} \cdot F_{bars_i} = 279.2344 \text{ kip ft}$$

Depth of each wire

$$\text{for } i := 1, i \le 13, i := i+1 \quad d_{wires_i} := \frac{\left| m \cdot x_{wires_{i\,1}} - x_{wires_{i\,2}} + b \right|}{\sqrt{m^2 + 1}}$$

Strain of each wire

$$\text{for } i := 1, i \le 13, i := i+1 \quad \varepsilon_{wires_i} := \frac{c - d_{wires_i}}{c} \cdot \varepsilon_{cu}$$

Stress in the concrete if wire is in compression

$$\text{for } i := 1, i \le 13, i := i+1 \quad \sigma_{c.wires_i} := \text{if } \varepsilon_{wires_i} < 0 \;\; 0 \text{ ksi} \;\; \text{else} \;\; f_c \cdot \frac{n \cdot \frac{\varepsilon_{wires_i}}{0.003}}{n - 1 + \left(\frac{\varepsilon_{wires_i}}{0.003} \right)^n}$$

Stress in each wire less concrete stress

$$\text{for } i := 1, i \le 13, i := i+1 \quad \sigma_{wires_i} := \max\left(\left[E_s \cdot \varepsilon_{wires_i} \;\; -f_m \right] \right) - \sigma_{c.wires_i}$$

Force in each wire

$$\text{for } i := 1, i \le 13, i := i+1 \quad F_{wires_i} := A_{wire} \cdot \sigma_{wires_i}$$

Total moment the mesh is generating about the x axis

$$M_{wx} := \sum_{i=1}^{13} x_{wires_{i\,2}} \cdot F_{wires_i} = -2.1602 \text{ kip ft}$$

Total moment the mesh is generating about the y axis

$$M_{wy} := \sum_{i=1}^{13} -x_{wires_{i\,1}} \cdot F_{wires_i} = 45.0754 \text{ kip ft}$$

Arrays of computed values using the above equations

$$d_{wires} = \begin{bmatrix} 5.231 \\ 6.2342 \\ 7.2374 \\ 8.2406 \\ 9.2438 \\ 10.247 \\ 11.2502 \\ 12.2534 \\ 13.2566 \\ 13.6955 \\ 9.8234 \\ 5.9512 \\ 2.0791 \end{bmatrix} \text{ in} \qquad \varepsilon_{wires} = \begin{bmatrix} 0.0002 \\ -0.0003 \\ -0.0009 \\ -0.0014 \\ -0.0019 \\ -0.0025 \\ -0.003 \\ -0.0036 \\ -0.0041 \\ -0.0043 \\ -0.0023 \\ -0.0002 \\ 0.0019 \end{bmatrix} \qquad \sigma_{c.wires} = \begin{bmatrix} 0.841 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 4.6589 \end{bmatrix} \text{ ksi}$$

$$\sigma_{wires} = \begin{bmatrix} 4.9558 \\ -9.7763 \\ -25.3494 \\ -40.9225 \\ -56.4956 \\ -65 \\ -65 \\ -65 \\ -65 \\ -65 \\ -65 \\ -5.3831 \\ 50.067 \end{bmatrix} \text{ ksi} \qquad F_{wires} = \begin{bmatrix} 0.1982 \\ -0.3911 \\ -1.014 \\ -1.6369 \\ -2.2598 \\ -2.6 \\ -2.6 \\ -2.6 \\ -2.6 \\ -2.6 \\ -2.6 \\ -0.2153 \\ 2.0027 \end{bmatrix} \text{ kip}$$

Total mild force $F_m := \left(\left(\sum F_{bars} \right) + \left(\sum F_{wires} \right) \right) = -129.7652 \text{ kip}$

Moment the mild reinforcement is generating about x axis $M_{xm} := M_{bx} + M_{wx} = 1.3082 \text{ kip ft}$

Moment the mild reinforcement is generating about y axis $M_{ym} := M_{by} + M_{wy} = 324.3098 \text{ kip ft}$

Convergance Check

Solution is correct when sum of forces is 0 $\Sigma F := F_c + F_m = -0.019 \text{ kip}$

Resistance Factor

Maximum strain in the reinforcement $\varepsilon_{max} := \max \left(\left[\max \left(-\varepsilon_{bars} \right) \ \max \left(-\varepsilon_{wires} \right) \right] \right) = 0.00466$

Yield strain of the rebar $\varepsilon_{ty} := \dfrac{f_y}{E_s} = 0.00207$

Resistance factor pre ACI 318-14 Table 21.2.2

$$\phi := \text{if } \varepsilon_{max} \le \varepsilon_{ty} \quad 0.65 \quad \text{else} \quad \text{if } \varepsilon_{max} \ge 0.005 \quad 0.90 \quad \text{else} \quad 0.65 + 0.25 \cdot \left(\frac{\varepsilon_{max} - \varepsilon_{ty}}{0.005 - \varepsilon_{ty}} \right) = 0.8708$$

Flexure Capacity

Nominal flexure capacity about y axis $M_{ny} := M_{yc} + M_{ym} = -0.0732$ kip ft

Flexure capacity about y axis $\phi M_{ny} := \phi \cdot M_{ny} = -0.0637$ kip ft

Nominal flexure capacity about x axis $M_{nx} := M_{xc} + M_{xm} = 119.7266$ kip ft

Flexure capacity about x axis $\phi M_{nx} := \phi \cdot M_{nx} = 104.2542$ kip ft