Transformed Section Properties

Description Calculation of transformed section properties using first principles. For this example the topping, rebar, and strand will all be transformed.

Concrete Geometry and Material Properties

Thickness of topping	$t_{top} := 2.75\ in$
Width of topping	$w_{top} := 68\ in$
Height of precast section	$h_{pre} := 32\ in$
Width of stem	$w_{stem} := 24\ in$
Thickness of ledge	$t_{ledge} := 12\ in$
Width of ledge	$w_{ledge} := 8\ in$
Elastic modulus of precast	$E_{pre} := 4695\ ksi$
Elastic modulus of topping	$E_{top} := 3834\ ksi$

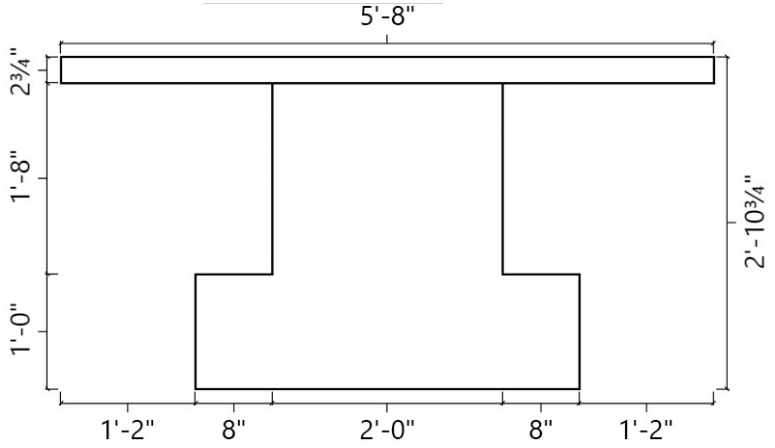

Figure 1: Concrete Geometry of Composite Precast IT Beam

Reinforcement Quantities and Properties

Number of strand in rows 1-4	$num_{row1} := 16$
	$num_{row2} := 14$
	$num_{row3} := 2$
	$num_{row4} := 2$
Centroid of strand in rows 1-4 measured from the bottom	$cg_{row1} := 3\ in$
	$cg_{row2} := 5\ in$
	$cg_{row3} := 10\ in$
	$cg_{row4} := 30\ in$
Number of bars	$num_{bars} := 4$
Centroid of bars measured from the bottom	$cg_{bars} := 29\ in$

Area of individual strand $A_{strand} := 0.167\ in^2$

Area of individual rebar $A_{rebar} := 1\ in^2$

Elastic modulus of mild reinforcement $E_s := 29000\ ksi$

Elastic modulus of prestressed reinforcement $E_{ps} := 29000\ ksi$

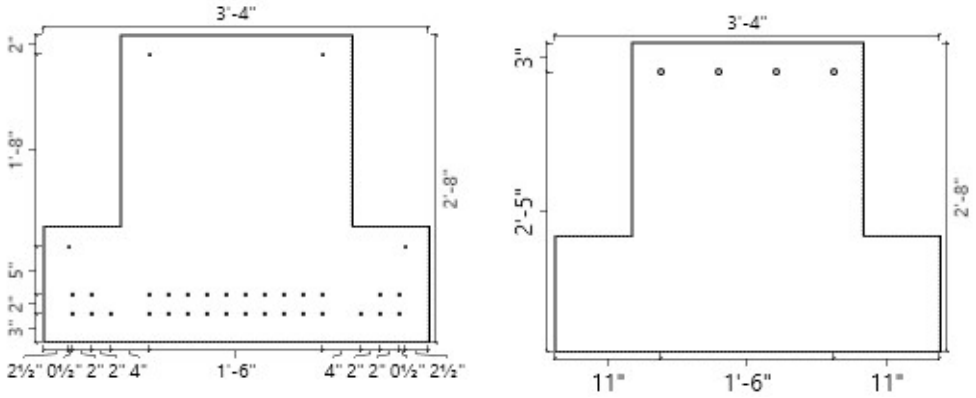

Figure 2: Reinforcement Locations of Strand (Left) and Rebar (Right)

Precast Section Properties

Area of precast secttion $A_{pre} := h_{pre} \cdot w_{stem} + 2 \cdot w_{ledge} \cdot t_{ledge} = 960\ in^2$

Centroid of precast section $cg_{y.pre} := \dfrac{h_{pre} \cdot w_{stem} \cdot \dfrac{h_{pre}}{2} + \left(2 \cdot \left(w_{ledge} \cdot t_{ledge} \cdot \dfrac{t_{ledge}}{2} \right) \right)}{A_{pre}} = 14\ in$

Moment of inertia of precast section $I_{xx.pre} := \dfrac{w_{stem} \cdot h_{pre}^3}{12} + h_{pre} \cdot w_{stem} \cdot \left(\dfrac{h_{pre}}{2} - cg_{y.pre} \right)^2 + 2 \cdot \left(\dfrac{w_{ledge} \cdot t_{ledge}^3}{12} + w_{ledge} \cdot t_{ledge} \cdot \left(\dfrac{t_{ledge}}{2} - cg_{y.pre} \right)^2 \right) = 83200\ in^4$

Topping Section Properties

Modular ratio of topping compared to precast $\eta_{top} := \dfrac{E_{top}}{E_{pre}} = 0.8166$

Transformed topping area $A_{top} := \eta_{top} \cdot t_{top} \cdot w_{top} = 152.7067\ in^2$

Transformed topping centroid $cg_{y.top} := h_{pre} + \dfrac{t_{top}}{2} = 33.375\ in$

Transformed topping moment of inertia $I_{xx.top} := \eta_{top} \cdot \dfrac{1}{12} \cdot w_{top} \cdot t_{top}^3 = 96.237\ in^4$

Composite Section Properties

Area of composite section $A_{comp} := A_{pre} + A_{top} = 1112.7067\ in^2$

Centroid of composite section $cg_{y.comp} := \dfrac{A_{pre} \cdot cg_{y.pre} + A_{top} \cdot cg_{y.top}}{A_{comp}} = 16.659\ in$

Moment of inertia of composite section

$$I_{xx.comp} := I_{xx.pre} + A_{pre} \cdot \left(cg_{y.pre} - cg_{y.comp} \right)^2 + = 132753.7\ in^4$$
$$+ I_{xx.top} + A_{top} \cdot \left(cg_{y.top} - cg_{y.comp} \right)^2$$

Transformed Section Properties

Moduluar ratio of rebar

$$\eta_{rebar} := \frac{E_s}{E_{pre}} - 1 = 5.1768$$

Transformed area of rebar

$$A_{rebar} := \eta_{rebar} \cdot num_{bars} \cdot A_{rebar} = 20.7071\ in^2$$

Moduluar ratio of strand

$$\eta_{strand} := \frac{E_{ps}}{E_{pre}} - 1 = 5.1768$$

Tranformed area of strand row 1-4

$$A_{strand.row1} := \eta_{strand} \cdot num_{row1} \cdot A_{strand} = 13.8324\ in^2$$

$$A_{strand.row2} := \eta_{strand} \cdot num_{row2} \cdot A_{strand} = 12.1033\ in^2$$

$$A_{strand.row3} := \eta_{strand} \cdot num_{row3} \cdot A_{strand} = 1.729\ in^2$$

$$A_{strand.row4} := \eta_{strand} \cdot num_{row4} \cdot A_{strand} = 1.729\ in^2$$

Total transformed area of strand

$$A_{strand} := A_{strand.row1} + A_{strand.row2} + A_{strand.row3} + A_{strand.row4} = 29.3938\ in^2$$

Area of transformed section

$$A_{trans} := A_{pre} + A_{top} + A_{rebar} + A_{strand} = 1162.8076\ in^2$$

Strand component of centroid calculation

$$cg_{y.strand} := A_{strand.row1} \cdot cg_{row1} + A_{strand.row2} \cdot cg_{row2} + \quad = 171.1755\ in^3$$
$$+ \left(A_{strand.row3} \cdot cg_{row3} + A_{strand.row4} \cdot cg_{row4} \right)$$

Centroid of transformed section

$$cg_{y.trans} := \frac{A_{pre} \cdot cg_{y.pre} + A_{top} \cdot cg_{y.top} + A_{rebar} \cdot cg_{bars} + cg_{y.strand}}{A_{trans}} = 16.6049\ in$$

Strand component of moment of inertia calculation

$$I_{xx.strand} := A_{strand.row1} \cdot \left(cg_{y.trans} - cg_{row1} \right)^2 + A_{strand.row2} \cdot \left(cg_{y.trans} - cg_{row2} \right)^2 + \quad = 4575.9276\ in^4$$
$$+ \left(A_{strand.row3} \cdot \left(cg_{y.trans} - cg_{row3} \right)^2 + A_{strand.row4} \cdot \left(cg_{y.trans} - cg_{row4} \right)^2 \right)$$

Moment of inertia of transformed section

$$I_{xx.trans} := I_{xx.pre} + A_{pre} \cdot \left(cg_{y.trans} - cg_{y.pre} \right)^2 + I_{xx.top} + \quad = 140514.344\ in^4$$
$$+ A_{top} \cdot \left(cg_{y.trans} - cg_{y.top} \right)^2 + \left(A_{rebar} \cdot \left(cg_{y.trans} - cg_{bars} \right)^2 + I_{xx.strand} \right)$$