Cracked Section Properties

Reference	Mast, Robert F. "Analysis of Cracked Prestressed Concrete Sections: A Practical Approach." PCI Design Handbook 8th Edition
Description	Calculation of the cracked section properties outlined by Robert Mast in the PCI Journal and Design Handbook. Delta fps is also computed once transformed properties are determined. A summary of the procedure can be found in the Cracked-Section Analysis section in section 5.2.2.2 of the PCI Design Handbook.

Concrete Geometry and Material Properties

Thickness of topping	$t_{top} := 2.75\ in$
Width of topping	$w_{top} := 68\ in$
Height of precast section	$h_{pre} := 32\ in$
Width of stem	$w_{stem} := 24\ in$
Elevation of centroid	$cg_{gb} := 14\ in$
Depth of centroid	$cg_{gt} := h_{pre} - cg_{gb} = 18\ in$
Gross Section Moment of Inertia	$I_{zz.g} := 83200\ in^4$
Thickness of ledge	$t_{ledge} := 12\ in$
Width of ledge	$w_{ledge} := 8\ in$
Elastic modulus of precast	$E_{pre} := 4695\ ksi$
Elastic modulus of topping	$E_{top} := 3834\ ksi$
Gross Cross Section Area	$A_g := 960\ in^2$
Depth of the crack	$d_{crack} := 19.84381103515625\ in$

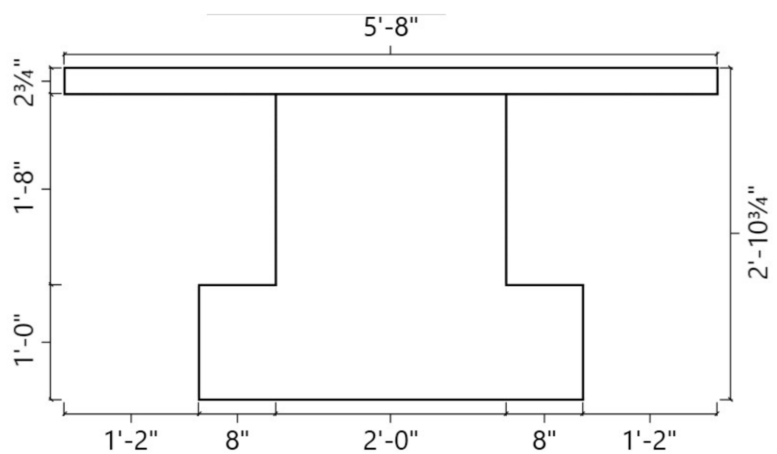

Figure 1: Concrete Geometry of Composite Precast IT Beam

Reinforcement Quantities and Properties

Number of strand in rows 1-4

$num_{row1} := 16$

$num_{row2} := 14$

$num_{row3} := 2$

$num_{row4} := 2$

Centroid of strand in rows 1-4 measured from the bottom

$cg_{row1} := 3\ in$

$cg_{row2} := 5\ in$

$cg_{row3} := 10\ in$

$cg_{row4} := 30\ in$

Number of bars $num_{bars} := 4$

Centroid of bars measured from the bottom $cg_{bars} := 29\ in$

Area of individual strand $A_{strand} := 0.167\ in^2$

Area of individual rebar $A_{rebar} := 1\ in^2$

Total area of strand

$$A_{ps} := A_{strand} \cdot \left(num_{row1} + num_{row2} + num_{row3} + num_{row4}\right) = 5.678\ in^2$$

Centroid of strand

$$cg_{st} := \frac{num_{row1} \cdot cg_{row1} + num_{row2} \cdot cg_{row2} + num_{row3} \cdot cg_{row3} + num_{row4} \cdot cg_{row4}}{num_{row1} + num_{row2} + num_{row3} + num_{row4}} = 5.824\ in$$

Total area of rebar $A_s := A_{rebar} \cdot num_{bars} = 4\ in^2$

Ultimate stress of strand $f_{pu} := 270\ ksi$

Elastic modulus of mild reinforcement $E_s := 29000\ ksi$

Elastic modulus of prestressed reinforcement $E_{ps} := 29000\ ksi$

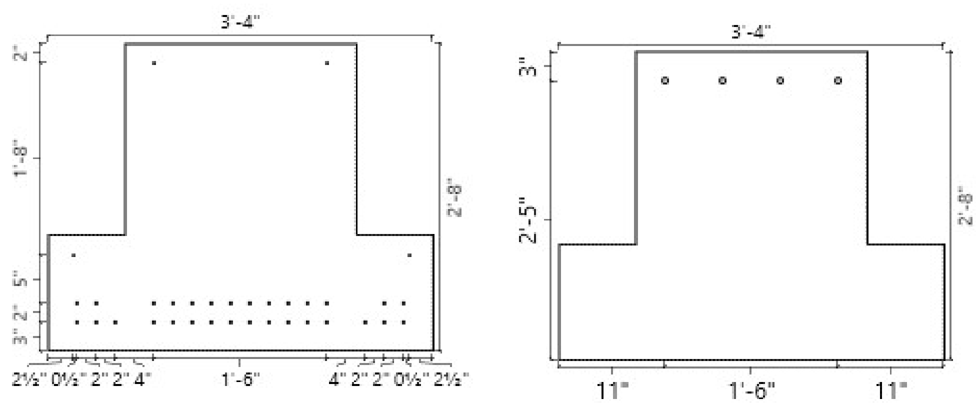

Figure 2: Reinforcement Locations of Strand (Left) and Rebar (Right)

Loading

Moment due to self weight $M_{SW} := 249.389\ kip\ ft$

Moment due to non-composite dead load $M_{Dnc} := 498.778\ kip\ ft$

Moment due to topping weight $M_{Dt} := 48.5789\ kip\ ft$

Moment due to composite dead load $M_{Dc} := 997.556\ kip\ ft$

Moment due to live load $M_{LL} := 748.167\ kip\ ft$

Total moment on the non composite section $M_{nc} := M_{SW} + M_{Dnc} + M_{Dt} = 796.7459\ kip\ ft$

Total moment due to external forces $M_{ext} := M_{SW} + M_{Dnc} + M_{Dt} + M_{Dc} + M_{LL} = 2542.4689\ kip\ ft$

Decompression Stress and Force in Strand

Stresses at centroid of strand due to:

Self weight and prestress $f_{cir} := 1615.346\ \text{psi}$

All other sustained loads $f_{cds} := 1622.554\ \text{psi}$

Prestress losses $Loss := 8.4814\ \%$

Initial stress in strand after losses $f_{se} := 0.75 \cdot f_{pu} \cdot (1 - Loss) = 185.3252\ \text{ksi}$

Decompression stress of the strand $f_{dc} := f_{se} + \left(f_{cir} - f_{cds}\right) \cdot \dfrac{E_{ps}}{E_{pre}} = 185.2806\ \text{ksi}$

Decomptression force in the strand $P_{ps} := A_{ps} \cdot f_{dc} = 1052.0235\ \text{kip}$

Decomporession Stress and force in Bars

Creep term from prestress losses $CR := 0 \cdot psi$

Shrinkage term from prestress losses $SH := 4280.4\ \text{psi}$

Decompression stress in bars $f_{dc.bars} := -(CR + SH) = -4.2804\ \text{ksi}$

Decompression force in bars $P_s := A_s \cdot f_{dc.bars} = -17.1216\ \text{kip}$

Fictitious Topping Force

Strand eccentricity $e := cg_{gb} - cg_{st} = 8.1765\ \text{in}$

Prestressing force $P_i := A_{ps} \cdot f_{se} = 1052.2763\ \text{kip}$

Prestressing moment $M_i := P_i \cdot e = 8603.9061\ \text{kip in}$

Non composite loading stresses at:

Bottom of precast $f_{pb} := \dfrac{P_i}{A_g} + \dfrac{\left(M_i - M_{nc}\right) \cdot cg_{gb}}{I_{zz.g}} = 935.08\ \text{psi}$

Top of precst: $f_{pt} := \dfrac{P_i}{A_g} + \dfrac{\left(M_{nc} - M_i\right) \cdot cg_{gt}}{I_{zz.g}} = 1303.174\ \text{psi}$

Change in stress $\Delta f := f_{pt} - f_{pb} = 368.0941\ \text{psi}$

Height of the composite section $h_c := h_{pre} + t_{top} = 34.75\ \text{in}$

Fictitious force in top of topping $f_{t.fic} := f_{pb} + \Delta f \cdot \dfrac{h_c}{h_{pre}} = 1334.8072\ \text{psi}$

Average fictitious force in topping $f_{fic} := 0.5 \cdot \left(f_{pt} + f_{t.fic}\right) = 1318.9906\ \text{psi}$

Moduluar ratio $\eta := \dfrac{E_{top}}{E_{pre}} = 0.8166$

Ficitious topping force $P_t := \eta \cdot w_{top} \cdot t_{top} \cdot f_{fic} = 201.4187\ \text{kip}$

Cemtrpod of fictitious topping force

$$e_{pt} := h_{pre} + \frac{\eta \cdot w_{top} \cdot t_{top} \cdot \left(f_{pt} \cdot \dfrac{t_{top}}{2} + \left(0.5 \cdot \left(f_{t.fic} - f_{pt}\right) \cdot \dfrac{2}{3} \cdot t_{top}\right)\right)}{P_t} = 33.3805\ \text{in}$$

Total load Pe

$$P_e := P_s + P_{ps} + P_t = 1236.3206\ \text{kip}$$

Location of load Pe

$$cg_{Pe} := \frac{P_{ps} \cdot cg_{st} + P_s \cdot cg_{bars} + P_t \cdot e_{pt}}{P_e} = 9.9921\ \text{in}$$

Precast Section Properties

Area of uncracked precast secttion

$$A_{pre} := \left(h_{pre} - d_{crack}\right) \cdot w_{stem} = 291.7485\ \text{in}^2$$

Centroid of precast section

$$cg_{y.pre} := d_{crack} + \left(\frac{h_{pre} - d_{crack}}{2}\right) = 25.9219\ \text{in}$$

Moment of inertia of precast section

$$I_{xx.pre} := \frac{w_{stem} \cdot \left(h_{pre} - d_{crack}\right)^3}{12} = 3592.7113\ \text{in}^4$$

Topping Section Properties

Modular ratio of topping compared to precast

$$\eta_{top} := \frac{E_{top}}{E_{pre}} = 0.8166$$

Transformed topping area

$$A_{top} := \eta_{top} \cdot t_{top} \cdot w_{top} = 152.7067\ \text{in}^2$$

Transformed topping centroid

$$cg_{y.top} := h_{pre} + \frac{t_{top}}{2} = 33.375\ \text{in}$$

Transformed topping moment of inertia

$$I_{xx.top} := \eta_{top} \cdot \frac{1}{12} \cdot w_{top} \cdot t_{top}^{\ 3} = 96.237\ \text{in}^4$$

Composite Section Properties

Area of composite section

$$A_{comp} := A_{pre} + A_{top} = 444.4552\ \text{in}^2$$

Centroid of composite section

$$cg_{y.comp} := \frac{A_{pre} \cdot cg_{y.pre} + A_{top} \cdot cg_{y.top}}{A_{comp}} = 28.4827\ \text{in}$$

Moment of inertia of composite section

$$I_{xx.comp} := I_{xx.pre} + A_{pre} \cdot \left(cg_{y.pre} - cg_{y.comp}\right)^2 + = 9257.1\ \text{in}^4$$
$$+ I_{xx.top} + A_{top} \cdot \left(cg_{y.top} - cg_{y.comp}\right)^2$$

Transformed Section Properties

Moduluar ratio of rebar

$$\eta_{rebar} := \frac{E_s}{E_{pre}} - 1 = 5.1768$$

Transformed area of rebar

$$A_{rebar} := \eta_{rebar} \cdot num_{bars} \cdot A_{rebar} = 20.7071\ \text{in}^2$$

Moduluar ratio of strand

$$\eta_{strand} := \frac{E_{ps}}{E_{pre}} = 6.1768$$

Tranformed area of strand row 1-4

$$A_{strand.row1} := \eta_{strand} \cdot num_{row1} \cdot A_{strand} = 16.5044\ \text{in}^2$$

$$A_{strand.row2} := \eta_{strand} \cdot num_{row2} \cdot A_{strand} = 14.4413\ \text{in}^2$$

$$A_{strand.row3} := \eta_{strand} \cdot num_{row3} \cdot A_{strand} = 2.063\ \text{in}^2$$

$$A_{strand.row4} := \left(\eta_{strand} - 1\right) \cdot num_{row4} \cdot A_{strand} = 1.729\ in^2$$

Total transformed area of strand

$$A_{strand} := A_{strand.row1} + A_{strand.row2} + A_{strand.row3} + A_{strand.row4} = 34.7378\ in^2$$

Area of transformed section

$$A_{trans} := A_{pre} + A_{top} + A_{rebar} + A_{strand} = 499.9002\ in^2$$

Strand component of centroid calculation

$$cg_{y.strand} := A_{strand.row1} \cdot cg_{row1} + A_{strand.row2} \cdot cg_{row2} + \left(A_{strand.row3} \cdot cg_{row3} + A_{strand.row4} \cdot cg_{row4}\right) = 194.2215\ in^3$$

Centroid of transformed section

$$cg_{y.trans} := \frac{A_{pre} \cdot cg_{y.pre} + A_{top} \cdot cg_{y.top} + A_{rebar} \cdot cg_{bars} + cg_{y.strand}}{A_{trans}} = 26.9134\ in$$

Strand component of moment of inertia calculation

$$I_{xx.strand} := A_{strand.row1} \cdot \left(cg_{y.trans} - cg_{row1}\right)^2 + A_{strand.row2} \cdot \left(cg_{y.trans} - cg_{row2}\right)^2 + \left(A_{strand.row3} \cdot \left(cg_{y.trans} - cg_{row3}\right)^2 + A_{strand.row4} \cdot \left(cg_{y.trans} - cg_{row4}\right)^2\right) = 16979.2883\ in^4$$

Moment of inertia of transformed section

$$I_{xx.trans} := I_{xx.pre} + A_{pre} \cdot \left(cg_{y.trans} - cg_{y.pre}\right)^2 + I_{xx.top} + A_{top} \cdot \left(cg_{y.trans} - cg_{y.top}\right)^2 + \left(A_{rebar} \cdot \left(cg_{y.trans} - cg_{bars}\right)^2 + I_{xx.strand}\right) = 27421.1121\ in^4$$

Convergance Check

Internal moment

$$M_{int} := M_{ext} - P_e \cdot \left(cg_{y.trans} - cg_{Pe}\right) = 799.1255\ kip\ ft$$

Stress at neutral axis (should be 0)

$$f_{na} := \left(\frac{P_e}{A_{trans}} + M_{int} \cdot \frac{\left(d_{crack} - cg_{y.trans}\right)}{I_{xx.trans}}\right) = 0.826\ psi$$

Delta Fps

Modulus ratio of strand

$$\eta := \frac{E_{ps}}{E_{pre}} = 6.1768$$

Increase in stress at centroid of strand from cracking

$$\Delta f_{ps} := \eta \cdot \left(\frac{P_e}{A_{trans}} + M_{int} \cdot \frac{\left(cg_{st} - cg_{y.trans}\right)}{I_{xx.trans}}\right) = -30280.1\ psi$$