Uniaxial Flexure Capacity

Description	Calculation of flexure capacity use strain compatibility of a section bending uniaxially. Concrete stress strain curve is from Collins and Mitchell.
Reference	PCI Design Handbook 8th Edition| ACI 318-14 Collins, Michael P. and Mitchell, Denis, Prestressed Concrete Structures, Prentice Hall, Englewood Cliffs, NJ, 1991

Concrete Geometry and Material Properties

Thickness of topping $t_{top} := 2.75\ in$

Width of topping $w_{top} := 68\ in$

Height of precast section $h_{pre} := 32\ in$

Width of stem $w_{stem} := 24\ in$

Elastic modulus of precast $E_{pre} := 4695\ ksi$

Elastic modulus of topping $E_{top} := 3834\ ksi$

Compressive strength of the topping $f_{ct} := 4\ ksi$

Compressive strength of the precast $f_{cp} := 6\ ksi$

Ultimate strain in concrete $\varepsilon_{cu} := 0.003$

Constant used for stress strain curve of concrete

$$n := \frac{E_{pre}}{E_{pre} - \dfrac{f_{cp}}{0.003}} = 1.7421$$

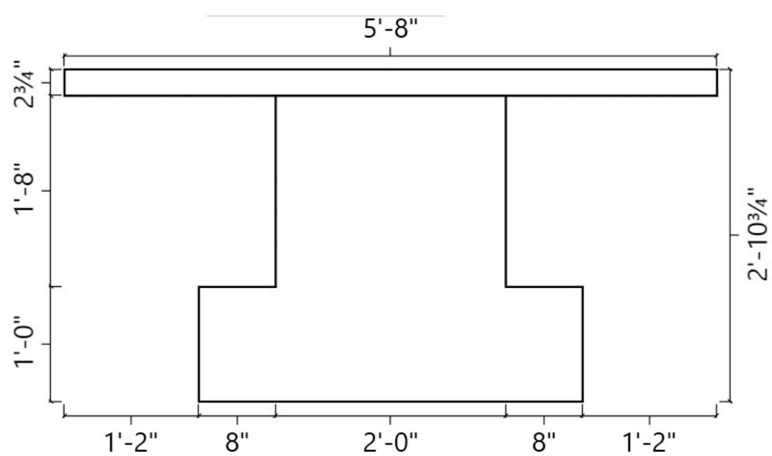

Figure 1: Concrete Geometry of Composite Precast IT Beam

Reinforcement Quantities and Properties

Number of strand in rows 1-4

$num_{row1} := 16$

$num_{row2} := 14$

$num_{row3} := 2$

$num_{row4} := 2$

Centroid of strand in rows 1-4 measured from the bottom

$cg_{row1} := 3\ in$

$cg_{row2} := 5\ in$

$cg_{row3} := 10\ in$

$cg_{row4} := 30\ in$

Area of individual strand $A_{strand} := 0.167\ in^2$

Jacking ratio for strand $Jacking := 0.75$

Prestress losses $Losses := 8.48\ \%$

Ultimate stress in strand $f_{pu} := 270\ ksi$

Elastic modulus of prestressed reinforcement $E_{ps} := 29000\ ksi$

Centroid of bars measured from the bottom $cg_{bars} := 29\ in$

Number of bars $num_{bars} := 4$

Area of individual rebar $A_{rebar} := 1\ in^2$

Total area of mild reinforcement $A_s := A_{rebar} \cdot num_{bars} = 4\ in^2$

Elastic modulus of mild reinforcement $E_s := 29000\ ksi$

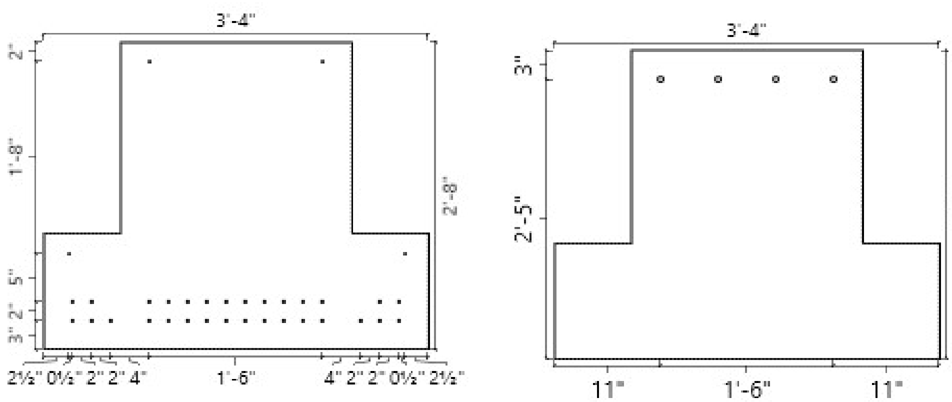

Figure 2: Reinforcement Locations of Strand (Left) and Rebar (Right)

Flexure Parameters

Neutral axis depth $c := 10.32775\ in$

Area of topping in above neutral axis $A_{top} := w_{top} \cdot t_{top} = 187\ in^2$

Area of precast in above neutral axis $A_{pre} := w_{stem} \cdot \left(c - t_{top}\right) = 181.866\ in^2$

Average concrete strength in compression zone $f_{ca} := \dfrac{A_{top} \cdot f_{ct} + A_{pre} \cdot f_{cp}}{A_{top} + A_{pre}} = 5\ ksi$

Beta factor $\beta_1 := 0.85 - 0.05 \cdot \dfrac{\left(f_{ca} - 4000\ psi\right)}{1000\ psi} = 0.801$

Concrete Force

Depth of compression block $a := \beta_1 \cdot c = 8.2694\ in$

Area of topping in compression block $A_{top} := w_{top} \cdot t_{top} = 187\ in^2$

Area of precast in compression block $A_{pre} := w_{stem} \cdot \left(a - t_{top}\right) = 132.4653\ in^2$

Compression force in topping $F_{ct} := 0.85 \cdot f_{ct} \cdot A_{top} = 635.8\ \text{kip}$

Compression force in precast $F_{cp} := 0.85 \cdot f_{cp} \cdot A_{pre} = 675.573\ \text{kip}$

Compression force in concrete $F_c := F_{ct} + F_{cp} = 1311.373\ \text{kip}$

Centroid of concrete force

$$cg_c := \frac{F_{ct} \cdot \left(h_{pre} + \frac{t_{top}}{2}\right) + F_{cp} \cdot \left(h_{pre} - \left(\frac{a - t_{top}}{2}\right)\right)}{F_c} = 31.245\ \text{in}$$

Mild Force

Depth of mild reinforcement $d_{bars} := h_{pre} + t_{top} - cg_{bars} = 5.75\ \text{in}$

Strain in mild reinforcement

$$\varepsilon_{bars} := \frac{c - d_{bars}}{c} \cdot \varepsilon_{cu} = 0.001330$$

Stress in concrete the rebar is replacing

$$\sigma_c := f_{cp} \cdot \frac{n \cdot \frac{\varepsilon_{bars}}{0.003}}{n - 1 + \left(\frac{\varepsilon_{bars}}{0.003}\right)^n} = 4.7063\ \text{ksi}$$

Stress in rebar less concrete stress $\sigma_{bars} := E_s \cdot \varepsilon_{bars} - \sigma_c = 33.8562\ \text{ksi}$

Total mild force $F_m := \sigma_{bars} \cdot A_s = 135.4249\ \text{kip}$

Centroid of mild force $cg_m := cg_{bars} = 29\ \text{in}$

Strand Force

Depths of strand rows 1-4

$$d_{s1} := h_{pre} + t_{top} - cg_{row1} = 31.75\ \text{in}$$

$$d_{s2} := h_{pre} + t_{top} - cg_{row2} = 29.75\ \text{in}$$

$$d_{s3} := h_{pre} + t_{top} - cg_{row3} = 24.75\ \text{in}$$

$$d_{s4} := h_{pre} + t_{top} - cg_{row4} = 4.75\ \text{in}$$

Initial strain in strand

$$\varepsilon_i := \left(-\frac{Jacking \cdot f_{pu}}{E_{ps}}\right) \cdot (1 - Losses) = -6.391 \cdot 10^{-3}$$

Strain in strand rows 1-4 due to flexure

$$\varepsilon_{s1} := \frac{c - d_{s1}}{c} \cdot \varepsilon_{cu} = -6.223 \cdot 10^{-3}$$

$$\varepsilon_{s2} := \frac{c - d_{s2}}{c} \cdot \varepsilon_{cu} = -5.6418 \cdot 10^{-3}$$

$$\varepsilon_{s3} := \frac{c - d_{s3}}{c} \cdot \varepsilon_{cu} = -4.1894 \cdot 10^{-3}$$

$$\varepsilon_{s4} := \frac{c - d_{s4}}{c} \cdot \varepsilon_{cu} = 0.0016$$

Stress in concrete the strand is replacing

$$\sigma_{c1} := 0\ \text{ksi}$$

$$\sigma_{c2} := 0\ \text{ksi}$$

$$\sigma_{c3} := 0\ \text{ksi}$$

$$\sigma_{c4} := f_{cp} \cdot \frac{n \cdot \frac{\varepsilon_{s4}}{0.003}}{n - 1 + \left(\frac{\varepsilon_{s4}}{0.003}\right)^{n}} = 5.2077 \text{ ksi}$$

Stress strain relationship from PCI Design Aid 15.2.3.

The two curves that make up this curve are not continuous using the values they use as constants. Because of this, a new constant is computed to ensure continuity.

Yield stress of the strand $$f_{py} := 0.9 \cdot f_{pu} = 243 \text{ ksi}$$

Yield strand of the strand $$\varepsilon_{yp} := \frac{f_{py}}{E_{ps}} = 0.0083793103$$

Recomputed constant $$c_1 := \varepsilon_{yp} - \frac{0.04 \text{ ksi}}{f_{pu} - E_{ps} \cdot \varepsilon_{yp}} = 0.007$$

Stress in strand rows less concrete stress

$$\sigma_{s1} := -\text{if } -\left(\varepsilon_i + \varepsilon_{s1}\right) \le \varepsilon_{yp} \;\; \left(-\left(\varepsilon_i + \varepsilon_{s1}\right)\right) \cdot E_{ps} \;\; \text{else} \;\; f_{pu} - \frac{0.04}{-\left(\varepsilon_i + \varepsilon_{s1}\right) - c_1} \text{ ksi} \; + \sigma_{c1} = -263.0015 \text{ ksi}$$

$$\sigma_{s2} := -\text{if } -\left(\varepsilon_i + \varepsilon_{s2}\right) \le \varepsilon_{yp} \;\; \left(-\left(\varepsilon_i + \varepsilon_{s2}\right)\right) \cdot E_{ps} \;\; \text{else} \;\; f_{pu} - \frac{0.04}{-\left(\varepsilon_i + \varepsilon_{s2}\right) - c_1} \text{ ksi} \; + \sigma_{c2} = -262.2097 \text{ ksi}$$

$$\sigma_{s3} := -\text{if } -\left(\varepsilon_i + \varepsilon_{s3}\right) \le \varepsilon_{yp} \;\; \left(-\left(\varepsilon_i + \varepsilon_{s3}\right)\right) \cdot E_{ps} \;\; \text{else} \;\; f_{pu} - \frac{0.04}{-\left(\varepsilon_i + \varepsilon_{s3}\right) - c_1} \text{ ksi} \; + \sigma_{c3} = -259.1368 \text{ ksi}$$

$$\sigma_{s4} := -\text{if } -\left(\varepsilon_i + \varepsilon_{s4}\right) \le \varepsilon_{yp} \;\; \left(-\left(\varepsilon_i + \varepsilon_{s4}\right)\right) \cdot E_{ps} \;\; \text{else} \;\; f_{pu} - \frac{0.04}{-\left(\varepsilon_i + \varepsilon_{s4}\right) - c_1} \text{ ksi} \; + \sigma_{c4} = -133.1339 \text{ ksi}$$

Force in strand rows 1-4

$$F_{s1} := num_{row1} \cdot A_{strand} \cdot \sigma_{s1} = -702.74 \text{ kip}$$

$$F_{s2} := num_{row2} \cdot A_{strand} \cdot \sigma_{s2} = -613.0462 \text{ kip}$$

$$F_{s3} := num_{row3} \cdot A_{strand} \cdot \sigma_{s3} = -86.5517 \text{ kip}$$

$$F_{s4} := num_{row4} \cdot A_{strand} \cdot \sigma_{s4} = -44.4667 \text{ kip}$$

Total force in strand $$F_s := F_{s1} + F_{s2} + F_{s3} + F_{s4} = -1446.8046 \text{ kip}$$

Centroid of strand force $$cg_s := \frac{F_{s1} \cdot cg_{row1} + F_{s2} \cdot cg_{row2} + F_{s3} \cdot cg_{row3} + F_{s4} \cdot cg_{row4}}{F_s} = 5.096 \text{ in}$$

Convergance Check

Solution is correct when sum of forces is 0

$$\Sigma F := F_c + F_m + F_s = -0.007 \text{ kip}$$

Resistance Factor

Maximum strain in the reinforcement

$$\varepsilon_{max} := \max\left(\begin{bmatrix} -\varepsilon_{bars} & -\varepsilon_{s1} & -\varepsilon_{s2} & -\varepsilon_{s3} & -\varepsilon_{s4} \end{bmatrix}\right) = 0.0062$$

Limit train for compression controlled

$$\varepsilon_{ty} := 0.002$$

Resistance factor pre ACI 318-14 Table 21.2.2

$$\phi := \begin{cases} 0.65 & \text{if } \varepsilon_{max} \le \varepsilon_{ty} \\ 0.90 & \text{else if } \varepsilon_{max} \ge 0.005 \\ 0.65 + 0.25 \cdot \left(\dfrac{\varepsilon_{max} - \varepsilon_{ty}}{0.005 - \varepsilon_{ty}}\right) & \text{else} \end{cases} = 0.9$$

Flexure Capacity

Nominal flexure capacity

$$M_n := F_c \cdot cg_c + F_m \cdot cg_m + F_s \cdot cg_s = 3127.345 \text{ kip ft}$$

Flexure capacity

$$\phi M_n := \phi \cdot M_n = 2814.6105 \text{ kip ft}$$