Shear Capacity - Prestressed

References ACI 318-14

Description Calculation of shear capacity for a prestressed beam using ACI 318-14. The shear capacity is calculated two different ways By both accounting for a cast-in-place topping and by ignoring it and relying only on the precast section

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Concrete:

Precast Strength	$f_c := 5\ ksi$
Toppinng Strength	$f_{ct} := 4\ ksi$

Section Information:

Precast Height	$h_{pre} := 21\ in$
Topping Thickness	$h_{topping} := 3\ in$
Web Thickness	$b_w := 24\ in$
Transverse Yield	$f_{yt} := 60\ ksi$
Gross Moment of Inertia	$I_{zz.g} := 22441.04\ in^4$
Composite Moment of Inertia	$I_{zz.c} := 33426.2857\ in^4$
Centroid Height	$cg_y := 9.46\ in$
Centroid Height	$cg_{comp.y} := 10.857\ in$
Precast Area	$A_g := 600\ in^2$

Prestress:

Strand Count	$num_{strand} := 14$
Area of Prestress	$A_{ps} := num_{strand} \cdot 0.153\ in^2 = 2.142\ in^2$
Prestress Losses	$losses := 15\ \%$
Composite Reinforcement Depth	$d := 18\ in$
Prestress Force	$P_i := A_{ps} \cdot 0.70 \cdot 270\ ksi \cdot (1 - losses) = 344.1123\ kip$
Eccentricity	$e := \left(cg_y - \left(h_{pre} - d\right)\right) = 6.46\ in$

Loading Information:

Vertical Shear	$V_u := 244\ kip$
Shear from Dead Load	$V_d := 150\ kip$
Ultimate Moment	$M_u := 83\ kip\ ft$
Moment from Dead Load	$M_d := 12\ kip\ ft$

Method 1: Precast Only

The cast-in-place topping on the member will be ignored. The precast concrete strength and geoemtry will be used when computing all values.

Concrete Shear Strength

Externally applied shear force

$$V_i := V_u - V_d = 94 \text{ kip}$$

Externally applied moment

$$M_{max} := M_u - M_d = 71 \text{ kip ft}$$

Effective Reinforcement Depth

$$d_{eff} := \max\left(\begin{bmatrix} d - h_{topping} & 0.8 \cdot h_{pre} \end{bmatrix}\right) = 16.8 \text{ in}$$

Stress from Dead Load

$$f_d := \frac{M_d \cdot cg_y}{I_{zz.g}} = 60.7 \text{ psi}$$

Stress from prestress

$$f_{pe} := \left(\frac{P_i}{A_g} + \frac{P_i \cdot e \cdot cg_y}{I_{zz.g}}\right) = 1510.6 \text{ psi}$$

External Cracking Moment
(Eq. 22.5.8.3.1c)

$$M_{cre} := \frac{I_{zz.c}}{cg_{comp.y}} \cdot \left(6 \cdot \sqrt{\frac{f_c}{\text{psi}}} \text{ psi} + f_{pe} - f_d\right) = 5770.2 \text{ kip in}$$

Shear Capacity from Flexure + Shear
(Eq. 22.5.8.3.1a)

$$V_{ci.a} := 0.6 \cdot \sqrt{\frac{f_c}{\text{psi}}} \text{ psi} \cdot b_w \cdot d_{eff} + V_d + \frac{V_i \cdot M_{cre}}{M_{max}} = 803.7 \text{ kip}$$

(Eq. 22.5.8.3.1b)

$$V_{ci.b} := 1.7 \cdot \sqrt{\frac{f_c}{\text{psi}}} \text{ psi} \cdot b_w \cdot d_{eff} = 48.4679 \text{ kip}$$

Sec 22.5.8.3.1 states to use the maximum of the previous two equations.

$$V_{ci} := \max\left(\begin{bmatrix} V_{ci.a} & V_{ci.b} \end{bmatrix}\right) = 803.7196 \text{ kip}$$

Noncomposite Compression Stress

$$f_{pc} := \frac{P_i}{A_g} - \frac{\left(P_i \cdot e - M_d\right) \cdot \left(cg_{comp.y} - cg_y\right)}{I_{zz.g}} = 444.1 \text{ psi}$$

Shear Capacity from Web Tension
(Eq. 22.5.8.3.2)

$$V_{cw} := \left(3.5 \cdot \sqrt{\frac{f_c}{\text{psi}}} \text{ psi} + 0.3 \cdot f_{pc}\right) \cdot b_w \cdot d_{eff} = 153.5 \text{ kip}$$

Concrete Shear Capacity

$$V_c := \min\left(\begin{bmatrix} V_{ci} & V_{cw} \end{bmatrix}\right) = 153.5 \text{ kip}$$

Required Transverse Reinforcement

Required Transverse Contribution

$$V_{s.req} := \frac{V_u}{0.75} - V_c = 171.828 \text{ kip}$$

Required Transverse Area per Foot
(Eq. 22.5.10.5.3)

$$A_{v.req} := \frac{V_{s.req}}{f_{yt} \cdot d_{eff}} = 2.0456 \frac{\text{in}^2}{\text{ft}}$$

Minimum Transverse Reinforcement
(Table 9.6.3.3)

$$A_{v.min.a} := 0.75 \cdot \sqrt{\frac{f_c}{\text{psi}}} \text{ psi} \cdot \frac{b_w}{f_{yt}} = 0.2546 \frac{\text{in}^2}{\text{ft}}$$

$$A_{v.min.b} := \frac{A_{ps} \cdot 270 \text{ ksi}}{80 \cdot f_{yt} \cdot d_{eff}} \cdot \sqrt{\frac{d_{eff}}{b_w}} = 0.072 \frac{\text{in}^2}{\text{ft}}$$

$$A_{v.min} := \min\left(\begin{bmatrix} A_{v.min.a} & A_{v.min.b} \end{bmatrix}\right) = 0.072 \frac{\text{in}^2}{\text{ft}}$$

Method 2: Full Section using Minimum Concrete Strength

For this method the two things that are changing at d, which is now considering the full section height, and the concrete strength, which is now using the minimum of the precast's material properties and the topping's material properties. Most terms, like fpc and Mcre, remain unchanged.

Concrete Shear Strength

Effective Reinforcement Depth

$$d_{eff} := \max\left(\begin{bmatrix} 0.8 \cdot \left(h_{pre} + h_{topping}\right) \\ d \end{bmatrix}\right) = 19.2\ \text{in}$$

Lightweight Concrete Factor

$$\lambda := 0.75$$

Shear Capacity from Flexure + Shear (Eq. 22.5.8.3.1a)

$$V_{ci} := 0.6 \cdot \lambda \cdot \sqrt{\frac{f_{ct}}{\text{psi}}}\ \text{psi} \cdot b_w \cdot d_{eff} + V_d + \frac{V_i \cdot M_{cre}}{M_{max}} = 799.7278\ \text{kip}$$

Shear Capacity from Web Tension (Eq. 22.5.8.3.2)

$$V_{cw} := \left(3.5 \cdot \lambda \cdot \sqrt{\frac{f_{ct}}{\text{psi}}}\ \text{psi} + 0.3 \cdot f_{pc}\right) \cdot b_w \cdot d_{eff} = 137.8943\ \text{kip}$$

Concrete Shear Capacity

$$V_c := \min\left(\begin{bmatrix} V_{ci} & V_{cw} \end{bmatrix}\right) = 137.8943\ \text{kip}$$

Required Transverse Reinforcement

Required Transverse Contribution

$$V_{s.req} := \frac{V_u}{0.75} - V_c = 187.439\ \text{kip}$$

Required Transverse Area per Foot (Eq. 22.5.10.5.3)

$$A_{v.req} := \frac{V_{s.req}}{f_{yt} \cdot d_{eff}} = 1.9525\ \frac{\text{in}^2}{\text{ft}}$$