6" Riser Deflection Verification

References	"ACI 318 14" "Notes of ACI 318 08 Building Code Requirements for Structural Concrete"
Design File	6" Riser.ebf
Description	Verification of Eriksson Beam's current deflection calculation and an additional deflection computation following the procedure outlined in Example 10.2 contained with PCA Notes on ACI 318

Section Geometry and Section Properties

Length $L = 13.75\ \text{ft}$

Width $w = 4\ \text{ft}$

Height $h = 6\ \text{in}$

Moment of Inertia $I_{xx} = \frac{1}{12} \cdot w \cdot h^3 = 864\ \text{in}^4$

Area $A_g = w \cdot h = 288\ \text{in}^2$

Reinforcement Information

Area of Mild Steel $A_s = 6 \cdot 0.31\ \text{in}^2 = 1.86\ \text{in}^2$

Depth to Mild Steel $d = 4.4375\ \text{in}$

Material Properties

Steel Elastic Modulus $E_s = 29000\ \text{ksi}$

Concrete Weight $w_c = 150\ \frac{\text{lbf}}{\text{ft}^3}$

Initial Concrete Elastic Modulus $E_{ci} = 3031.24\ \text{ksi}$

Final Concrete Elastic Modulus $E_c = 4695.98\ \text{ksi}$

Final Concrete Strength $f_c = 6\ \text{ksi}$

Modulus of Rupture $f_r = 7.5 \cdot \sqrt{\frac{f_c}{\text{psi}}}\ \text{psi} = 580.9475\ \text{psi}$

Modular Ratio $\eta = \frac{E_s}{E_c} = 6.1755$

Loading and Demand

Self Weight $w_{sw} = A_g \cdot w_c = 300\ \frac{\text{lbf}}{\text{ft}}$

Superimposed Dead Load $w_{sd} = 72\ \frac{\text{lbf}}{\text{ft}^2} \cdot w = 288\ \frac{\text{lbf}}{\text{ft}}$

Live Load $w_l = 100\ \frac{\text{lbf}}{\text{ft}^2} \cdot w = 400\ \frac{\text{lbf}}{\text{ft}}$

Total Dead Load $w_d = w_{sw} + w_{sd} = 0.588 \ \frac{\text{kip}}{\text{ft}}$

Total Loading $w_t = w_d + w_l = 0.988 \ \frac{\text{kip}}{\text{ft}}$

Dead Load Moment $M_d = \frac{w_d \cdot L^2}{8} = 13.8961 \ \text{kip ft}$

Live Load Moment $M_l = \frac{w_l \cdot L^2}{8} = 9.4531 \ \text{kip ft}$

Total Moment $M_t = M_d + M_l = 23.3492 \ \text{kip ft}$

Cracked Section Properties

$$\rho = \frac{A_s}{w \cdot d} = 0.0087$$

$$k = \sqrt{2 \cdot \rho \cdot \eta + (\rho \cdot \eta)^2} - \rho \cdot \eta = 0.2789$$

$$c = k \cdot d = 1.2375 \ \text{in}$$

$$I_{cr} = w \cdot c \cdot (0.5 \cdot c)^2 + \frac{w \cdot c^3}{12} + \eta \cdot A_s \cdot (d - c)^2 = 147.9428 \ \text{in}^4$$

Effective Moment of Inertia

Cracking Moment $M_{cr} = \frac{f_r \cdot I_{xx}}{0.5 \cdot h} = 13.9427 \ \text{kip ft}$

Effective Moment of Inertia from dead load

$$I_{e.d} = \min\left(\left[I_{xx} \ \left(\left(\frac{M_{cr}}{M_d} \right)^3 \cdot I_{xx} + \left(1 - \left(\frac{M_{cr}}{M_d} \right)^3 \right) \cdot I_{cr} \right) \right]\right) = 864 \ \text{in}^4$$

Effective Moment of inertia from all loads

$$I_{e.t} = \left(\frac{M_{cr}}{M_t} \right)^3 \cdot I_{xx} + \left(1 - \left(\frac{M_{cr}}{M_t} \right)^3 \right) \cdot I_{cr} = 300.4095 \ \text{in}^4$$

Additional Deflection Multiplier $\lambda_\Delta = 2$

Deflections based on example 10.2 from PCA Notes

The main changes here are that it appears additional deflections due to creep and shrinkage are only based on the elastic response (similiar to how was are handling prestressed beams). So this term is computed seperately. Because of this, the deflection due to creep is isolated as it's own term.

Deflection due to self weight

$$\Delta_{sw} = \frac{5 \cdot w_{sw} \cdot L^4}{384 \cdot E_{ci} \cdot I_{e.d}} = 0.0921 \text{ in}$$

Deflection due to super-imposed dead load

$$\Delta_{sd} = \frac{5 \cdot w_{sd} \cdot L^4}{384 \cdot E_{c} \cdot I_{e.d}} = 0.0571 \text{ in}$$

Deflection due to all dead load

$$\Delta_d = \Delta_{sw} + \Delta_{sd} = 0.1492 \text{ in}$$

Deflection due to all loading

$$\Delta_t = \frac{5 \cdot w_{sw} \cdot L^4}{384 \cdot E_{ci} \cdot I_{e.t}} + \frac{5 \cdot \left(w_t - w_{sw}\right) \cdot L^4}{384 \cdot E_c \cdot I_{e.t}} = 0.6572 \text{ in}$$

Deflection due to live load

$$\Delta_l = \Delta_t - \Delta_d = 0.508 \text{ in}$$

The deflection due to live load is computed indirectly. Here we are quantifying it as the total deflection minus the deflection due to dead load. It is defined this way because the increase in live load causes more deflection because the change to the moment of inertia.

Total long term deflection with creep

$$\Delta_{final} = \left(\lambda_\Delta + 1\right) \cdot \Delta_d + \Delta_l = 0.9556 \text{ in}$$

Alternatively, the equation can be rearranged to the following

Additional Deflection due to creep

$$\Delta_{cr} = \lambda_\Delta \cdot \Delta_d = 0.2984 \text{ in}$$

Total long term deflection with creep

$$\Delta_{final} = \Delta_{cr} + \Delta_t = 0.9556 \text{ in}$$

Composite deflections based on example 10.2 from PCA Notes

Toppin Geometry

Topping Thickness

$$t = 2 \text{ in}$$

Moment of Inertia

$$I_{xx.t} = \frac{1}{12} \cdot w \cdot t^3 = 32 \text{ in}^4$$

Area

$$A_t = w \cdot t = 96 \text{ in}^2$$

Material Properties

Concrete Weight

$$w_c = 150 \frac{\text{lbf}}{\text{ft}^3}$$

Topping Concrete Elastic Modulus

$$E_{ct} = 3834.25 \text{ ksi}$$

Topping Concrete Strength

$$f_{ct} = 6 \text{ ksi}$$

Topping Modular Ratio

$$\eta_t = \frac{E_{ct}}{E_c} = 0.8165$$

Composite Section Properties

Composite Area

$$A_c = A_g + \eta_t \cdot A_t = 366.3836\ in^2$$

Composite Centroid

$$c_{yc} = \frac{\frac{h}{2} \cdot A_g + \left(h + \frac{t}{2}\right) \cdot A_t \cdot \eta_t}{A_c} = 3.8558\ in$$

Composite Moment of Inertia

$$I_{xx.c} = I_{xx} + I_{xx.t} + A_g \cdot \left(\frac{h}{2} - c_{yc}\right)^2 + \eta_t \cdot A_t \cdot \left(h + \frac{t}{2} - c_{yc}\right)^2$$

$$I_{xx.c} = 1881.8295\ in^4$$

Additional Loading

Composite Dead Load

$$w_{dc} = 72\ \frac{lbf}{ft^2} \cdot w = 288\ \frac{lbf}{ft}$$

Total Non-Composite Loading

$$w_d = w_{sw} + w_{sd} = 0.588\ \frac{kip}{ft}$$

Total Sustained Loading

$$w_{sus} = w_d + w_{dc} = 0.876\ \frac{kip}{ft}$$

Total Loading

$$w_t = w_{sus} + w_l = 1.276\ \frac{kip}{ft}$$

Sustained Moment

$$M_{sus} = \frac{w_{sus} \cdot L^2}{8} = 20.7023\ kip\ ft$$

Total Moment

$$M_t = \frac{w_t \cdot L^2}{8} = 30.16\ kip\ ft$$

Effective Moment of Inertia

Non-Composite Cracking Moment

$$M_{cr} = \frac{f_r \cdot I_{xx}}{0.5 \cdot h} = 13.94\ kip\ ft$$

Composite Cracking Moment

$$M_{cr.c} = \frac{I_{xx.c}}{c_{yc}} \cdot f_r - M_d \cdot \left(\frac{\left(\frac{I_{xx.c}}{c_{yc}}\right)}{\left(\frac{I_{xx}}{0.5 \cdot h}\right)} - 1\right) = 13.98\ kip\ ft$$

Composite Cracked Moment of Inertia

(From software) $$I_{cr.c} = 324 \text{ in}^4$$

Sustained Effective Moment of Inertia

$$I_{e.sus} = \min\left(\left[I_{xx.c} \; \left(\left(\frac{M_{cr.c}}{M_{sus}}\right)^3 \cdot I_{xx.c} + \left(1 - \left(\frac{M_{cr.c}}{M_{sus}}\right)^3\right) \cdot I_{cr.c}\right)\right]\right) = 803.2 \text{ in}^4$$

Non-Composite Effective Moment of inertia

$$I_{e.nc.t} = \min\left(\left[I_{xx} \; \left(\left(\frac{M_{cr}}{M_d}\right)^3 \cdot I_{xx} + \left(1 - \left(\frac{M_{cr}}{M_d}\right)^3\right) \cdot I_{cr}\right)\right]\right) = 864 \text{ in}^4$$

Composite Effective Moment of inertia

$$I_{e.c.t} = \left(\frac{M_{cr.c}}{M_t}\right)^3 \cdot I_{xx.c} + \left(1 - \left(\frac{M_{cr.c}}{M_t}\right)^3\right) \cdot I_{cr.c} = 479.1 \text{ in}^4$$

Deflections

Deflection due to self weight, creep

$$\Delta_{sw.cr} = \frac{5 \cdot w_{sw} \cdot L^4}{384 \cdot E_{ci} \cdot I_{e.sus}} = 0.0991 \text{ in}$$

Deflection due to self weight, instantaneous

$$\Delta_{sw.i} = \frac{5 \cdot w_{sw} \cdot L^4}{384 \cdot E_{ci} \cdot I_{e.nc.t}} = 0.0921 \text{ in}$$

Deflection due to non-composite dead load, creep

$$\Delta_{sd.cr} = \frac{5 \cdot w_{sd} \cdot L^4}{384 \cdot E_c \cdot I_{e.sus}} = 0.0614 \text{ in}$$

Deflection due to non-composite dead load, instantaneous

$$\Delta_{sd.i} = \frac{5 \cdot w_{sd} \cdot L^4}{384 \cdot E_c \cdot I_{e.nc.t}} = 0.057 \text{ in}$$

Deflection due to composite dead load, creep

$$\Delta_{dc.cr} = \frac{5 \cdot w_{dc} \cdot L^4}{384 \cdot E_c \cdot I_{e.sus}} = 0.061 \text{ in}$$

Deflection due to composite dead load, instantaneous

$$\Delta_{dc.i} = \frac{5 \cdot w_{dc} \cdot L^4}{384 \cdot E_c \cdot I_{e.c.t}} = 0.103 \text{ in}$$

Deflection due to live load, instantaneous

$$\Delta_{l.i} = \frac{5 \cdot w_l \cdot L^4}{384 \cdot E_c \cdot I_{e.c.t}} = 0.143 \text{ in}$$

Total deflections, creep

$$\Delta_{t.cr} = \Delta_{sw.cr} + \Delta_{sd.cr} + \Delta_{dc.cr} = 0.222 \text{ in}$$

Total deflections, instantaneous

$$\Delta_{t.i} = \Delta_{sw.i} + \Delta_{sd.i} + \Delta_{dc.i} + \Delta_{l.i} = 0.395 \text{ in}$$

Total additional deflections

$$\Delta_{cr} = \Delta_{t.cr} \cdot \lambda_{\Delta} = 0.444 \text{ in}$$

Total long term deflection with creep

$$\Delta_{final} = \Delta_{cr} + \Delta_{t.i} = 0.839 \text{ in}$$

Total Deflections due to Load Cases

Total Self Weight

$$\Delta_{sw} = \Delta_{sw.i} + \lambda_{\Delta} \cdot \Delta_{sw.cr} = 0.29 \text{ in}$$

Total Non-Composite Dead Load

$$\Delta_{sd} = \Delta_{sd.i} + \lambda_{\Delta} \cdot \Delta_{sd.cr} = 0.18 \text{ in}$$

Total Composite Dead Load

$$\Delta_{dc} = \Delta_{dc.i} + \lambda_{\Delta} \cdot \Delta_{dc.cr} = 0.226 \text{ in}$$

Total Live Load

$$\Delta_l = \Delta_{l.i} = 0.143 \text{ in}$$