Deflections and Shortening

References	PCI Design Handbook, 8th Edition ACI 209
Design File	Deflection and Cracking Verification.ebf
Description	Calculation of deflections for a prestressed class C member. The member cracks under dead load to demonstrate how long term multipliers, which are only for elastic effects, are handled. Additionaly, the member shortening due to creep, shrinkage, and prestressing effects is also calculated.

Geometry

Beam Length $L = 45\ \mathrm{ft}$

Gross Cross Section Area $A_g = 960\ \mathrm{in}^2$

Composite Cross Section Area of Precast $A_c = 1124\ \mathrm{in}^2$

Area of the Topping $A_t = 2.75\ \mathrm{in} \cdot 68\ \mathrm{in} = 187\ \mathrm{in}^2$

Centroid Location of the Gross Cross Section $cg_y = 14\ \mathrm{in}$

Height of the member $h = 32\ \mathrm{in}$

Gross Section Moment of Inertia $I_{zz.g} = 83200\ \mathrm{in}^4$

Composite Section Moment of Inertia $I_{zz.c} = 132753.7\ \mathrm{in}^4$

Material Properties

Initial Elastic Modulus $E_{ci} = 3586\ \mathrm{ksi}$

30-Day Elastic Modulus $E_{cf} = 4695\ \mathrm{ksi}$

Relative Humidity $R_h = 70\ \%$

Concrete Weight $w_c = 150\ \dfrac{\mathrm{lbf}}{\mathrm{ft}^3}$

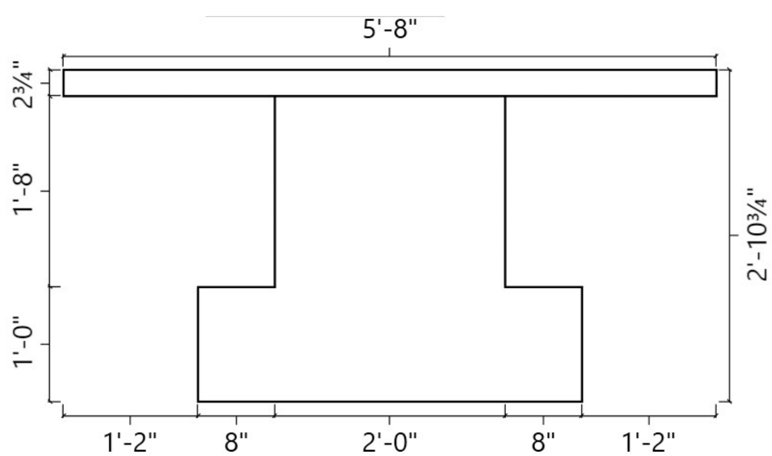

Figure 1: Concrete Geometry of Composite Precast IT Beam

Prestress Properties

Number of strand in rows 1-4 $num_{row1} = 16$

$num_{row2} = 14$

$num_{row3} = 2$

$num_{row4} = 2$

Centroid of strand in rows 1-4 measured from the bottom $cg_{row1} = 3\ in$

$cg_{row2} = 5\ in$

$cg_{row3} = 10\ in$

$cg_{row4} = 30\ in$

Number of bars $num_{bars} = 4$

Centroid of bars measured from the bottom $cg_{bars} = 29\ in$

Area of individual strand $A_{strand} = 0.167\ in^2$

Area of individual rebar $A_{rebar} = 1\ in^2$

Elastic modulus of mild reinforcement $E_s = 29000\ ksi$

Elastic modulus of prestressed reinforcement $E_{ps} = 29000\ ksi$

Total area of strand

$$A_{ps} = A_{strand} \cdot \left(num_{row1} + num_{row2} + num_{row3} + num_{row4}\right) = 5.678\ in^2$$

Centroid of strand

$$cg_{st} = \frac{num_{row1} \cdot cg_{row1} + num_{row2} \cdot cg_{row2} + num_{row3} \cdot cg_{row3} + num_{row4} \cdot cg_{row4}}{num_{row1} + num_{row2} + num_{row3} + num_{row4}} = 5.8235\ in$$

Initial Strand Stress $f_{pi} = 0.75 \cdot 270\ ksi = 202.5\ ksi$

Assumed Final Losses $Losses_{final} = 10\ \%$

Assumed Erection Losses $Losses_{construction} = 5\ \%$

Prestress Force $F_{ps} = A_{ps} \cdot f_{pi} \cdot \left(1 - Losses_{final}\right) = 1034.8155\ kip$

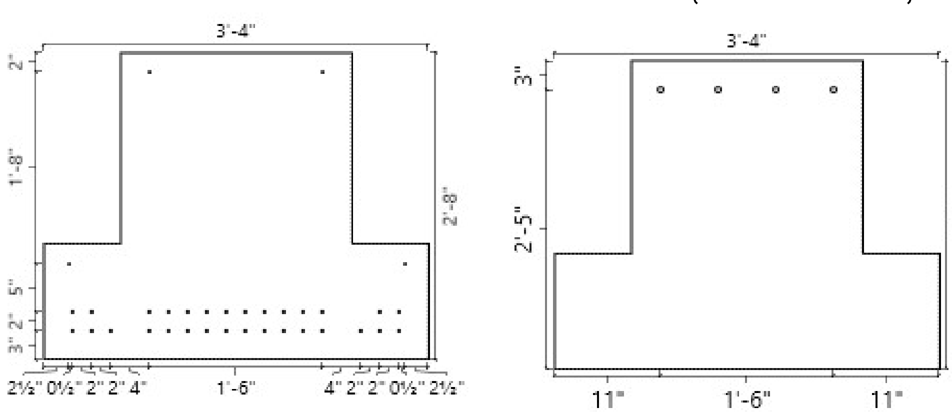

Figure 2: Reinforcement Locations of Strand (Left) and Rebar (Right)

Loading

Self Weight $w_{SW} = A_g \cdot w_c = 1.0000\ \frac{kip}{ft}$

Topping Weight $D_t = A_t \cdot w_c = 0.195\ \frac{kip}{ft}$

Non-Composite Dead Load $D_{nc} = 2\ \frac{kip}{ft}$

Composite Dead Load $D_c = 4\ \frac{kip}{ft}$

Live Load $LL = 3\ \frac{kip}{ft}$

Previously Computed Values

Cracking Moment $M_{cr} = 1586.81\ kip\ ft$

Cracked Section Moment of Inertia $I_{zz.cracked} = 25156.49\ in^4$

Volume to Surface Area Ratio $VS_{Ratio} = 6.6667$

Deflection Multipliers (PCI Table 5.9.2)

Self Weight Deflection Multiplier $\lambda_{Self.Weight} = 2.4$

Camber, Prestress, Deflection Multiplier $\lambda_{Camber} = 2.2$

Superimposed Dead Load Deflection Multiplier $\lambda_{Add'l.Dead} = 3.0$

Topping Deflection Multiplier $\lambda_{Topping} = 2.3$

Stage 1: Self Weight + Prestress

Assume the strand develops instantly and the moment due to prestress is constant.

Moment due to self weight $M_{SW} = \frac{w_{SW} \cdot L^2}{8} = 253.125\ kip\ ft$

Moment due to prestressing $M_{PS} = -\left(F_{ps} \cdot \left(cg_y - cg_{st}\right)\right) = -705.0949\ kip\ ft$

Total moment at the end of Stage 1 $M_T = M_{SW} + M_{PS} = -451.9699\ kip\ ft$

Deflections due self weight $\delta_{SW} = \lambda_{Self.Weight} \cdot \frac{5 \cdot w_{SW} \cdot L^4}{384 \cdot E_{ci} \cdot I_{zz.g}} = 0.742\ in$

Deflections due to prestress $\delta_{PS} = \lambda_{Camber} \cdot \frac{M_{PS} \cdot L^2}{8 \cdot E_{ci} \cdot I_{zz.g}} = -2.274\ in$

Total deflections at the end of Stage 1 $\delta_T = \delta_{SW} + \delta_{PS} = -1.532\ in$

Stage 2: Non-Composite Dead Load

Moment due to noncomposite dead load $M_{Dnc} = \frac{D_{nc} \cdot L^2}{8} = 506.25\ kip\ ft$

Total moment at the end of Stage 2 $M_T = M_T + M_{Dnc} = 54.28\ kip\ ft$

Deflections due to noncomposite dead load $\delta_{Dnc} = \lambda_{Add'l.Dead} \cdot \frac{5 \cdot D_{nc} \cdot L^4}{384 \cdot E_{cf} \cdot I_{zz.g}} = 1.417 \text{ in}$

Total deflections at the end of Stage 3 $\delta_T = \delta_T + \delta_{Dnc} = -0.1148 \text{ in}$

Stage 3: Topping Weight

Moment due to topping weight $M_{Dt} = \frac{D_t \cdot L^2}{8} = 49.3066 \text{ kip ft}$

Total moment at the end of Stage 3 $M_T = M_T + M_{Dt} = 1243.0412 \text{ kip in}$

Deflections due to topping $\delta_{Dt} = \lambda_{Topping} \cdot \frac{5 \cdot D_t \cdot L^4}{384 \cdot E_{cf} \cdot I_{zz.g}} = 0.106 \text{ in}$

Total deflections at the end of Stage 3 $\delta_T = \delta_T + \delta_{Dt} = -0.0089 \text{ in}$

Stage 4: Composite Dead Load

Moment due to composite dead load $M_{Dc} = \frac{D_c \cdot L^2}{8} = 1012.5 \text{ kip ft}$

Total moment at the end of Stage 4 $M_T = M_T + M_{Dc} = 1116.0868 \text{ kip ft}$

Crack check

```
if M_T - M_PS > M_cr = "Cracked"
  "Cracked"
else
  "Uncracked"
```

Percent over cracking moment $crack = \frac{M_T - M_{cr} - M_{PS}}{M_{Dc}} = 23.1 \%$

Percent under cracking moment $gross = 1 - crack = 76.9 \%$

Deflections due to the long term creep effects $\delta_{Dc.creep} = \left(\lambda_{Add'l.Dead} - 1\right) \cdot \frac{5 \cdot D_c \cdot L^4}{384 \cdot E_{cf} \cdot I_{zz.c}} = 1.184 \text{ in}$

Deflections due to loading on the uncracked section $\delta_{Dc.gross} = gross \cdot \frac{5 \cdot D_c \cdot L^4}{384 \cdot E_{cf} \cdot I_{zz.c}} = 0.455 \text{ in}$

Deflections due to loading on the cracked section $\delta_{Dc.cracked} = crack \cdot \frac{5 \cdot D_c \cdot L^4}{384 \cdot E_{cf} \cdot I_{zz.cracked}} = 0.723 \text{ in}$

Total deflection due to composite dead load $\delta_{Dc} = \delta_{Dc.creep} + \delta_{Dc.gross} + \delta_{Dc.cracked} = 2.363 \text{ in}$

Total deflections at the end of Stage 4 $\delta_T = \delta_T + \delta_{Dc} = 2.354 \text{ in}$

Stage 5: All Other Loads

Moment due to live load $M_{LL} = \frac{LL \cdot L^2}{8} = 759.375 \text{ kip ft}$

Total moment at the end of Stage 5 $M_T = M_T + M_{LL} = 1875.4618 \text{ kip ft}$

Deflection due to live load $\delta_{LL} = \frac{5 \cdot LL \cdot L^4}{384 \cdot E_{cf} \cdot I_{zz.cracked}} = 2.344 \text{ in}$

Total deflection at the end of 5tage 5 $\delta_T = \delta_T + \delta_{LL} = 4.6972 \text{ in}$

Shortening at Erection

The shortening calculation uses creep and shrinkage equations found in ACI 209. These equations are referenced below.

Prestress Force $F_{ps} = A_{ps} \cdot f_{pi} \cdot \left(1 - Losses_{construction}\right) = 1092.3053\ \text{kip}$

Moment due to prestressing $M_{PS} = -\left(F_{ps} \cdot \left(cg_y - cg_{st}\right)\right) = -744.2668\ \text{kip ft}$

End Rotation due to self weight $\theta_{SW} = -\dfrac{w_{SW} \cdot L^3}{24 \cdot E_{ci} \cdot I_{zz.g}} = -0.0018\ \text{rad}$

End Rotation due to prestress $\theta_{PS} = -\dfrac{M_{PS} \cdot L}{2 \cdot E_{ci} \cdot I_{zz.g}} = 0.0081\ \text{rad}$

Total end rotation at end of Stage 1 $\theta_T = \theta_{SW} + \theta_{PS} = 0.0062\ \text{rad}$

Base shortening $\delta_x = \dfrac{F_{ps} \cdot L}{A_g \cdot E_{ci}} = 0.1713\ \text{in}$

Time at erection $t = 30\ \text{day}$

Creep

Creep coefficient per ACI 209 $C_u = 2.35$ (Eq. A-19)

Reduction factor for volume to surface ratio $\phi_{cu.vs} = \dfrac{2}{3} \cdot \left(1 + 1.13 \cdot \mathbf{e}^{-0.54 \cdot VS_{Ratio}}\right) = 0.6873$ (Eq. A-25)

Reduction factor for relative humidity $\phi_{cu.Rh} = 1.27 - 0.67 \cdot R_h = 0.801$ (Eq. A-24)

Time factor $f_{tc} = \dfrac{\left(\dfrac{t}{\text{day}}\right)^{0.6}}{10 + \left(\dfrac{t}{\text{day}}\right)^{0.6}} = 0.4349$ (Eq. A-18)

Total creep multiplier $\gamma_{cr} = 1 + f_{tc} \cdot \phi_{cu.Rh} \cdot \phi_{cu.vs} \cdot C_u = 1.5626$ (Eq. A-21)

Shrinkage

Base shrinkage strain $\varepsilon_{SH} = 780 \cdot 10^{-6}$ (Eq. A-4)

Reduction factor for volume to surface ratio $\phi_{sh.vs} = 1.23 - 0.152 \cdot VS_{Ratio} = 0.2167$ (Eq. A-9)

Reduction factor for relative humidity $\phi_{sh.Rh} = 1.4 - 1.02 \cdot R_h = 0.686$ (Eq. A-7)

Time factor $f_{tsh} = \dfrac{\dfrac{t}{\text{day}}}{55 + \dfrac{t}{\text{day}}} = 0.3529$ (Eq. A-1)

Total shortening due to shrinkage $\delta_{sh} = f_{tsh} \cdot \phi_{sh.vs} \cdot \phi_{sh.Rh} \cdot \varepsilon_{SH} \cdot L = 0.0221\ \text{in}$

Total shortening at the CG of member	$\delta_{CG} = \delta_x \cdot \gamma_{cr} + \delta_{sh} = 0.2898\ in$
Bottom shortening due to flexure	$\delta_{flex.bot} = 2 \cdot cg_y \cdot \tan\left(\theta_T\right) = 0.175\ in$
Top shortening due to flexure	$\delta_{flex.top} = 2 \cdot \left(h - cg_y\right) \cdot \tan\left(-\theta_T\right) = -0.225\ in$
Total shortening at the CG	$\delta_{CG} = 0.2898\ in$
Total shortening at the bottom	$\delta_{CG} + \delta_{flex.bot} = 0.4648\ in$
Total shortening at the top	$\delta_{CG} + \delta_{flex.top} = 0.0648\ in$