Cracking Moment Hand Calculation

Description Calculation of cracking moment at midspan

Beam Geometry and Material Properties

Width of the precast and topping $w := 12\ ft$

Topping thickness $t_{top} := 4\ in$

Precast thickness $t_{pre} := 4\ in$

Lightweight concrete factor $\lambda := 0.85$

Concrete compression strength $f_c := 6\ ksi$

Modulus of rupture $f_r := 7.5 \cdot \lambda \cdot \sqrt{\frac{f_c}{psi}}\ psi = 493.8054\ psi$

Topping elastic modulus $E_{top} := 4286.83\ ksi$

Precast elastic modulus $E_{pre} := 3360.17\ ksi$

Moduluar ratio $\eta := \frac{E_{top}}{E_{pre}} = 1.2758$

Member length $L := 16\ ft + 8\ in = 200\ in$

Section Properties

Precast area $A_{g.pre} := t_{pre} \cdot w = 576\ in^2$

Topping area $A_{g.top} := t_{top} \cdot w = 576\ in^2$

Centroid of transformed composite section $cg_y := \frac{\eta \cdot A_{g.top} \cdot \left(t_{pre} + \frac{t_{top}}{2} \right) + A_{g.pre} \cdot \frac{t_{pre}}{2}}{\eta \cdot A_{g.top} + A_{g.pre}} = 4.2424\ in$

Moment of inertia of topping $I_{top} := \frac{w \cdot t_{top}^3}{12} = 768\ in^4$

Moment of inertia of precast $I_{pre} := \frac{w \cdot t_{pre}^3}{12} = 768\ in^4$

Moment of inertia of transformed composite section

$$I_{xx} := \eta \cdot \left(I_{top} + A_{g.top} \cdot \left(t_{pre} + \frac{t_{top}}{2} - cg_y \right)^2 \right) + I_{pre} + A_{g.pre} \cdot \left(\frac{t_{pre}}{2} - cg_y \right)^2 = 6914.1926\ in^4$$

Section modulus of non-composite section $S_b := \frac{I_{pre}}{0.5 \cdot t_{pre}} = 384\ in^3$

Section modulus of transformed composite section $S_{bc} := \frac{I_{xx}}{cg_y} = 1629.799\ in^3$

Prestressing Quantities

Area of prestress $A_{ps} := 12 \cdot 0.167\ \text{in}^2 = 2.004\ \text{in}^2$

Prestress losses $Loss := 11.11\ \%$

Jacking ratio $Pull := 75\ \%$

Ultimate strength of strand $f_{pu} := 270\ \text{ksi}$

Depth to strand $d_p := 6.75\ \text{in}$

Elevation of strand $cg_p := t_{pre} + t_{top} - d_p = 1.25\ \text{in}$

Eccentricity $e_p := 0.5 \cdot t_{pre} - cg_p = 0.75\ \text{in}$

Prestressing force $P_e := (1 - Loss) \cdot A_{ps} \cdot f_{pu} \cdot Pull = 360.7245\ \text{kip}$

Noncomposite Loading and Moment

Serive dead load factor $\gamma_{DL} := 1.0$

Self weight $w_{sw} := 480\ \frac{\text{lbf}}{\text{ft}}$

Topping weight $w_{dt} := 600\ \frac{\text{lbf}}{\text{ft}}$

Moment due to self weight $M_{sw} := \frac{\gamma_{DL} \cdot w_{sw} \cdot L^2}{8} = 200\ \text{kip in}$

Moment to to topping weight $M_{dt} := \frac{\gamma_{DL} \cdot w_{dt} \cdot L^2}{8} = 250\ \text{kip in}$

Factored noncomposite moment $M_{nc} := M_{sw} + M_{dt} = 450\ \text{kip in}$

Cracking Moment

Cracking moment $M_{cr} := S_{bc} \cdot \left(\frac{P_e}{A_{g.pre}} + \frac{P_e \cdot e_p}{S_b} + f_r \right) - M_{nc} \cdot \left(\frac{S_{bc}}{S_b} - 1 \right) = 126.1513\ \text{kip ft}$

1.2 * Cracking moment $1.2 \cdot M_{cr} = 151.3816\ \text{kip ft}$